Luminosity Distribution During Collider Run II

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1 Introduction

For Run II collider detector preparations it would be useful to correctly predict the luminosity and luminous region distributions before the start of Run II. Such a prediction is difficult to make since there are a large number of variables affecting the luminosity and extensive modifications are being made to almost every accelerator in the Fermilab Collider complex. Despite this we will present the best estimate of the expected luminosity during Collider Run II.

The expected performances of the Main Injector, Antiproton Source, Recycler, and Tevatron determine the estimated beam parameters and luminosity for Collider Run II. The assumptions, calculations, and experience forming these estimates are extensive [1] and not detailed in this memo. Instead, some of the more important factors in determining luminosity are presented as well as the impact these factors have on the luminosity and the luminosity distribution.

The memo lists the beam parameters and discusses the longitudinal luminosity distribution expected in Run II. A simplified version of the luminosity formula, given in the next section, is used to present the essential features of the Run II luminosity. Following this we list the expected Run II beam parameters along with typical Run I beam parameters. Experience from Run I is used to estimate the beam lifetimes and emittance growth rates expected during a Run II store. In the final sections we discuss the relation between the beam parameters and the length and width of the interaction region.

Appendix A is a detailed derivation of the formula used by the Beams Division to convert beam parameters into the luminosity. Appendix B contains statistics from Run I showing the variation of beam parameters from store to store and the beam lifetimes and emittance growth rates during a store.

2 Luminosity Formula

To simplify the discussions we use a simplified version of the luminosity formula as derived in Appendix A. The lattice functions are assumed to be the design values of $\beta^* = 35$ cm in both planes and zero dispersion in the interaction region. As presented in Appendix A, the particle density distributions are characterized as Gaussians in the longitudinal and transverse phase spaces. To simplify the discussion further we assume that the horizontal and vertical emittances of the protons are the same ($\epsilon_{1x} = \epsilon_{1y} = \epsilon_1$,) and that the horizontal and vertical emittances of the antiprotons are the same ($\epsilon_{2x} = \epsilon_{2y} = \epsilon_2$.) The proton and antiproton bunches will be colliding head-on (i.e. without a crossing angle) for the start of Run II with 36x36 bunch operations. Later in Run II, as we move to 132 nsec bunch spacing, the bunches will collide with a horizontal and vertical crossing angle, θ_x and θ_y, so we include the crossing angle in the luminosity formula.

The simplified formula for calculating the luminosity of a proton bunch (subscript 1) colliding with an antiproton bunch (subscript 2) is then

$$\mathcal{L} = \frac{f_{\rm rev} N_1 N_2}{2\pi\beta^*(\epsilon_1+\epsilon_2)} \frac{1}{\sqrt{2\pi}\sigma_z} \int \frac{1}{(1+z^2/\beta^{*2})} \exp[-\frac{z^2}{2\sigma_z^2} - \frac{2z^2}{\sigma_x^2}\theta_x^2 - \frac{2z^2}{\sigma_y^2}\theta_y^2]\, dz \qquad (1)$$

where

$$\begin{aligned}
\sigma_z^2 &= (\sigma_{1z}^2 + \sigma_{2z}^2)/4 \\
\sigma_x^2(z) &= \sigma_{1x}^2(z) + \sigma_{2x}^2(z) = (\epsilon_1+\epsilon_2)(\beta^* + \frac{z^2}{\beta^*}) \\
\sigma_y^2(z) &= \sigma_{1y}^2(z) + \sigma_{2y}^2(z) = (\epsilon_1+\epsilon_2)(\beta^* + \frac{z^2}{\beta^*})
\end{aligned}$$

In these formulas $f_{\rm rev}$ is the revolution frequency, N_1 is the number of protons in the bunch, ϵ_1 is the unnormalized emittance of the protons, σ_{1z} is the rms bunch length of the proton bunch, σ_{1x} and σ_{1y} are the transverse rms widths of the proton bunch. In Equation 1, and in this memo, the θ_x and θ_y are half the crossing angle in the horizontal and vertical planes. The emittances, ϵ, are un-normalized and related to the 95% normalized emittance, $\epsilon_{N,95\%}$, usually quoted at Fermilab via

$$\epsilon = \frac{\epsilon_{N,95\%}}{6\pi(\beta\gamma)_{\rm rel}} \qquad (2)$$

where $(\beta\gamma)_{\rm rel}$ is the relativistic $\beta\gamma$. Equation 1 applies for a single proton and antiproton bunch circulating in the Tevatron. In the case of 36x36 bunch operations Equation 1 is applied 36 times, once for each crossing.

The z coordinate in Equation 1 is the longitudinal coordinate (the axis along the beampipe) and it has been assumed that the center of the interaction region, the center of the detector, and the location of the minimum β function are all coincident at $z = 0$. In the form given, the shape of the longitudinal distribution of the luminosity is simply the integrand of Equation 1. This will be discussed further in later sections.

3 Collider Run II Beam Parameters

Based on the latest estimates from the Run II handbook, the expected beam parameters for Run II are listed in Table 1. This includes an estimate of the parameters near the beginning of Run II with 36x36 bunch operations and latter in Run II with 132 nsec bunch spacing and a crossing angle. For comparison the average beam parameters during a period of stable Run I operations are also included. (More details of the Run I parameters can be found in Appendix B.) Without giving detailed explanations of these estimates we make a few general comments distilled from the Run II Handbook.

The expected proton intensity and 95% normalized transverse emittance for Run II is about the same as was achieved during Run I. This is largely because the intensity of the protons bunches is limited by the beam-beam tune shift that the protons induce on the antiprotons. Previous collider experience, supported by calculations of the beam-beam tune shift, suggests that we will be operating near the beam-beam tune shift limit with 27×10^{10} protons per bunch. Thus there is little incentive to increase the proton intensity beyond the already established Run I levels. With the replacement of the Main Ring by the larger aperture and more efficient Main Injector, we anticipate no problems delivering this intensity of protons.

The longitudinal emittance of the protons delivered from the Main Injector in Run II is expected to be smaller than in Run I. This translates into a shorter proton bunch length at the start of a store as compared to Run I. The reduction in 95% normalized longitudinal emittance from about 5 eV-sec in Run I to about 2 eV-sec in Run II is expected because of the higher uncoalesced bunch intensity in the Main Injector. Since the same coalesced bunch intensity can be achieved with fewer uncoalesced bunches (about 5 instead of 11), the final longitudinal emittance of the coalesced bunches in the Main Injector will be smaller in Run II. It is assumed that the smaller emittance can be accelerated in the Tevatron to low beta without significant emittance blowup. Remember, this refers to the longitudinal emittance at the start of the store and the emittance will grow as the store progresses.

The antiproton intensity during Run II will be the parameter with the largest store to store variation, as it was during routine Run I operations. Because of Tevatron or Accumulator failures there was a large variation in the antiproton stack size at the starts of shot setups. This translated into a large variation in the antiproton bunch intensity since it is roughly proportional to the stack size. In addition to the variations in the stack size, the antiproton intensity will be affected by the performance of the upgrades to the Debuncher cooling system and the performance of the Recycler. Since our goal is to stack antiprotons at three times the Run I rate and since Fermilab has no experience with the Recycler in Collider operations it is difficult to predict the expected evolution of the antiproton intensity. For this reason we present several cases with differing antiproton bunch intensities.

The transverse emittance of the antiprotons is expected to be similar in Run II to what it was in Run I. The latest estimate for the 95% normalized antiproton emittance in Run II is 15π mm-mrad. We have used 20π mm-mrad in this memo because our calculations had been done with the previous estimate of 20π mm-mrad. Since this does not change our conclusions about the length of the luminous region we have not redone the calculations with the smaller emittance. There is some correlation between antiproton intensity and emittance but this effect is ignored since it needlessly complicates the issue. As with the protons, the longitudinal emittance of antiprotons is expected to be smaller due to the improved efficiency of the Main Injector.

Given the parameters at the start of a store for Collider Run II, and using models of emittance growth and beam lifetimes, we can make estimations of the evolution of luminosity during a store [1, 2]. The models to calculate beam lifetime and emittance growth rates include such effects as intrabeam scattering, particle loss due to particle interactions, and beam gas interactions. A comparison of this model to Run I data, shown in Appendix B, shows only a modest agreement. Nonetheless it is our best estimate and the one used for this memo.

Since the largest variation in operating parameters is expected to be the intensity of the antiproton bunches we present the evolution of beam parameters and luminosity for three different antiproton bunch intensities: 35, 70, and 100×10^9 antiprotons per bunch. Higher antiproton intensity results in higher transverse and longitudinal antiproton emittance growth rates. The evolution of beam parameters for these three cases are plotted in Figures 1–2.

As part of the store evolution model we use luminosity leveling to keep the average interaction rate below 5 interactions per crossing. In these plots the luminosity is intentionally reduced in the early part of a store by increasing the β^*. A plot of the β^* as a function of the store duration in shown in Figure 3.

RUN	Ib (1993-1995) (6×6)	Run II (36×36)	Run II (140×121)	
Protons/bunch	23	27	27	10^{10}
Proton Emittance	23π	20π	20π	mm-mrad
Proton Bunch Length	0.6	0.37	0.37	meter
Antiprotons/bunch	5.5	3.0	3.0	10^{10}
Antiproton bunches	6	36	121	
Total Antiprotons	33	110	360	10^{10}
Pbar Production Rate	6	20	20	10^{10}hr^{-1}
Antiproton Emittance†	13π	20π	20π	mm-mrad
Antiproton Bunch Length	0.6	0.37	0.37	meter
β^*	35	35	35	cm
Energy	900	1000	1000	GeV
Crossing Angle‡	0	0	136	μrad
Initial Luminosity	1.6	7.5	15	$\times 10^{31}\text{cm}^{-2}\text{s}^{-1}$
Luminosity per crossing	2.6	2.1	1.2	$\times 10^{30}\text{cm}^{-2}\text{s}^{-1}$
Interactions per Crossing	2.5	2.0	1.1	

Table 1: Comparison of measured Run I and expected Run II parameters. The emittances are 95% normalized. († The latest estimate for the Run II antiproton emittance is 15π, but the calculations in this memo were done for an emittance of 20π. This does not change the conclusions of luminous length significantly.) (‡ The crossing angle is 1/2 the angle between the proton and antiproton orbit in the horizontal and vertical plane. The total angle between the proton and antiproton orbit is 385 μrad.)

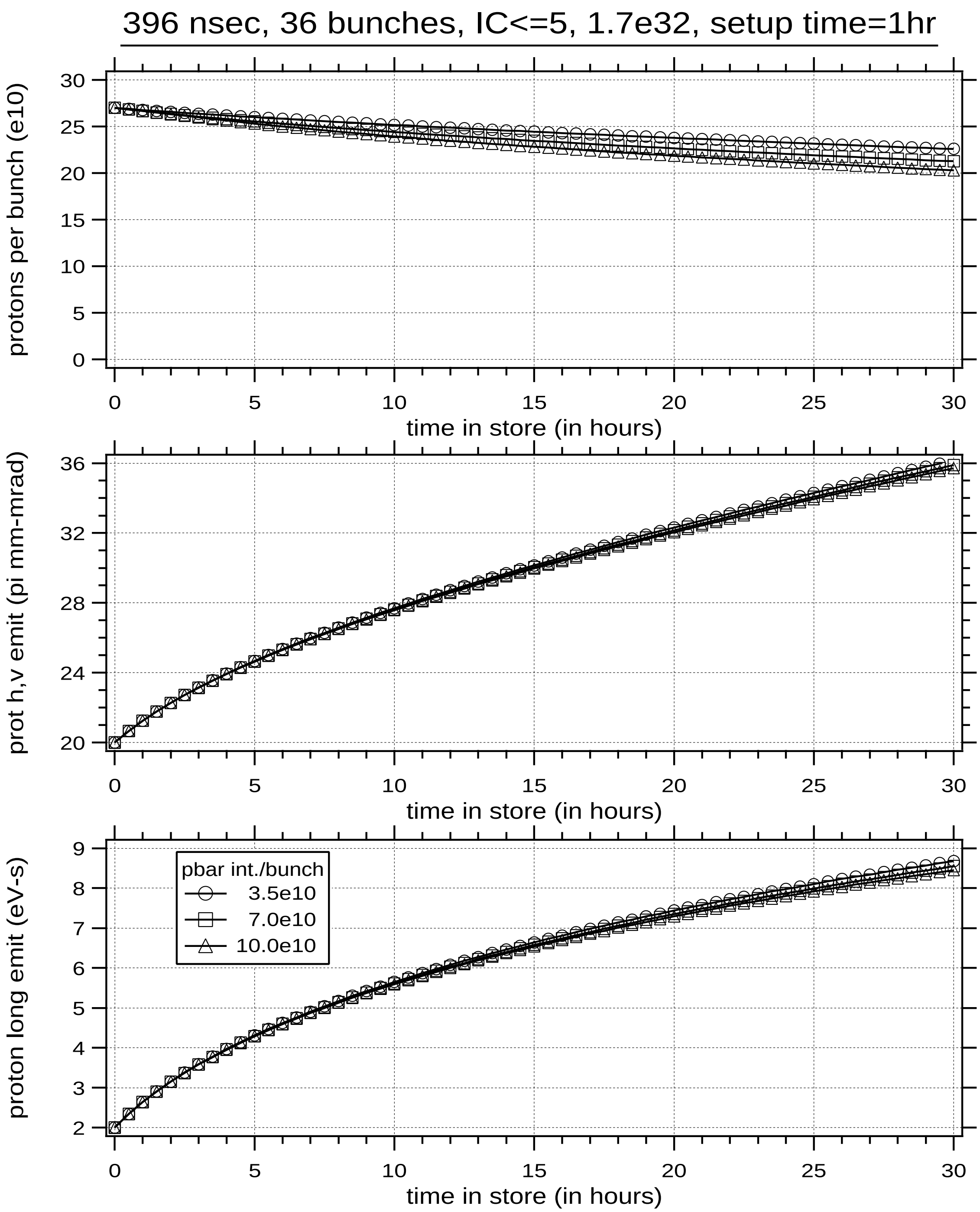

Figure 1: Estimated proton intensity, horizontal and vertical emittance, and longitudinal emittance during a Run II store. These are plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiprotons per bunch. Emittances are 95% normalized values.

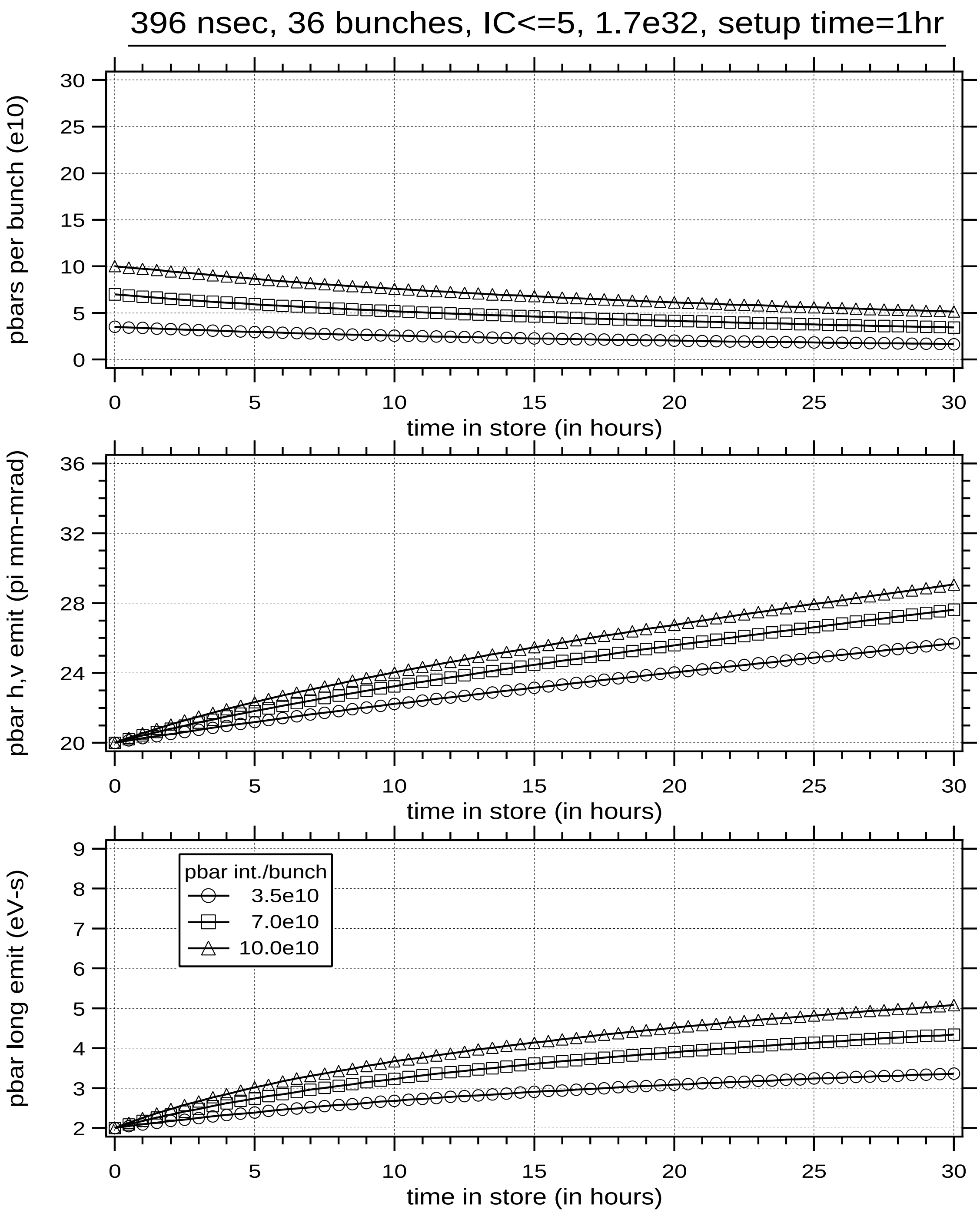

Figure 2: Estimated antiproton intensity, horizontal and vertical emittance, and longitudinal emittance during a Run II store. These are plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiprotons per bunch. Emittances are 95% normalized values.

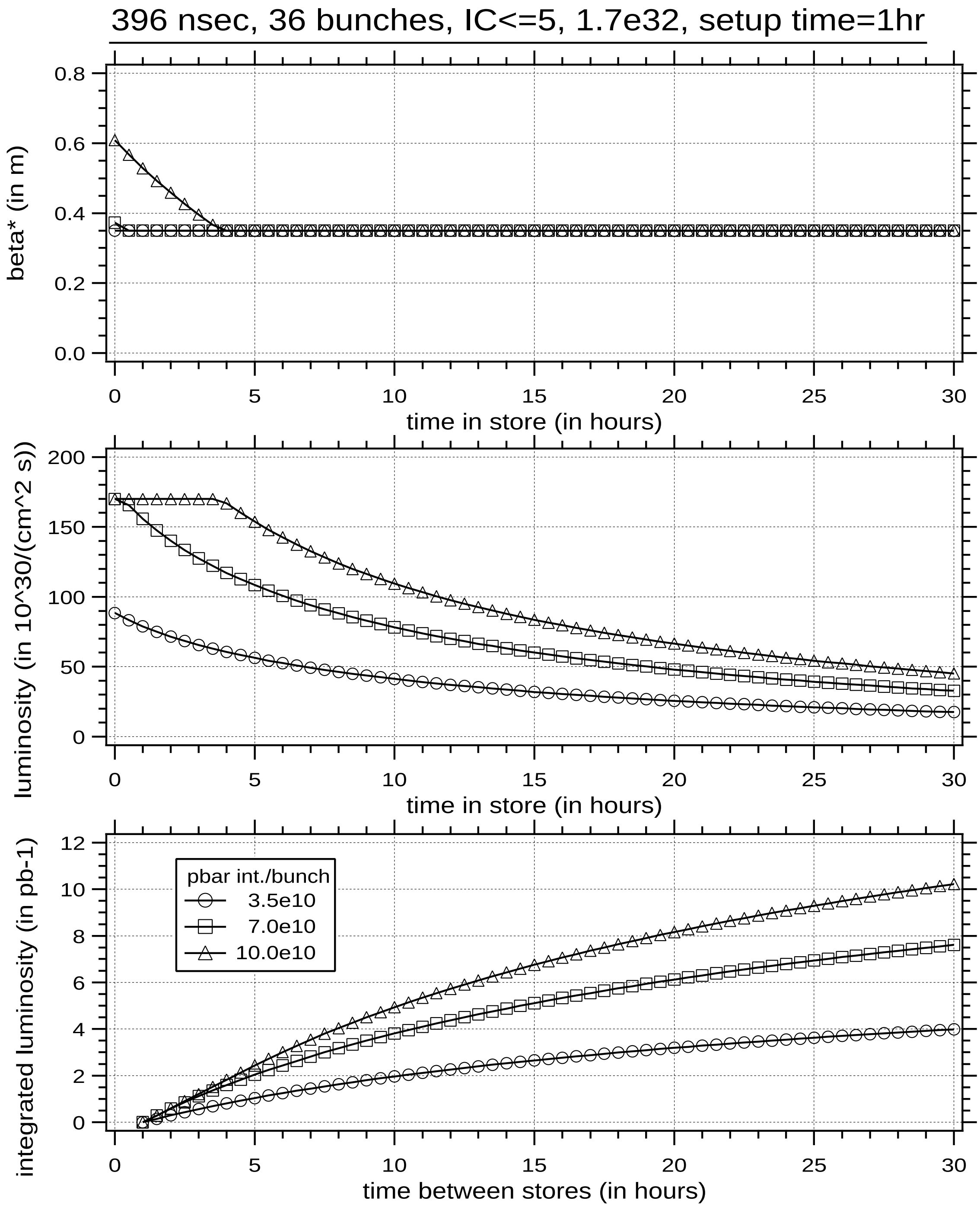

Figure 3: Calculated β^*, luminosity, and integrated luminosity during a Run II store. Luminosity leveling limits this to less than 5 interactions per crossing on average. These are plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiprotons per bunch. A one hour shot setup time has been used for the integrated luminosity calculation.

4 Longitudinal Luminosity Distribution

Although we cannot precisely predict the length of the luminous region we can still make some comments on the factors that contribute to the length of the luminous region. The longitudinal distribution of luminosity is given by the integrand of Equation 1 repeated below,

$$\mathcal{L} = \frac{f_{\mathrm{rev}} N_1 N_2}{2\pi\beta^*(\epsilon_1+\epsilon_2)} \frac{1}{\sqrt{2\pi}\sigma_z} \int \frac{1}{(1+z^2/\beta^{*2})} \exp[-\frac{z^2}{2\sigma_z^2} - \frac{2z^2}{\sigma_x^2}\theta_x^2 - \frac{2z^2}{\sigma_y^2}\theta_y^2]\, dz \tag{3}$$

where

$$\begin{aligned}
\sigma_z^2 &= (\sigma_{1z}^2 + \sigma_{2z}^2)/4 \\
\sigma_x^2(z) &= \sigma_{1x}^2(z) + \sigma_{2x}^2(z) = (\epsilon_1+\epsilon_2)(\beta^* + \frac{z^2}{\beta^*}) \\
\sigma_y^2(z) &= \sigma_{1y}^2(z) + \sigma_{2y}^2(z) = (\epsilon_1+\epsilon_2)(\beta^* + \frac{z^2}{\beta^*})
\end{aligned}$$

To prevent confusion we emphasize the factors that determine the longitudinal distribution. There are the proton and antiproton rms bunch lengths σ_{1z} and σ_{2z}. The bunch lengths are combined into the factor $\sigma_z^2 = (\sigma_{1z}^2+\sigma_{2z}^2)/4$ which is contained in the integral of Equation 1. (Without the hourglass effect and crossing angles, the longitudinal rms length of the luminous region would equal σ_z.)

The hourglass effect is another factor in the length of the interaction region. The transverse beam profile in the interaction region is shaped like an hourglass. The larger beam width (and hence lower particle density) on the ends of the interaction region reduces the number of interactions occurring at the ends of the interaction region. This hourglass effect is reflected in the $(1+z^2/\beta^{*2})^{-1}$ factor in the integrand of the luminosity. Note that the proton and antiproton emittances are not a factor in the length of the luminous region when there is no crossing angle.

The final factor we consider is the effect of a crossing angle. Adding a crossing angle reduces the length of the luminous region by separating the proton and antiproton orbits at the ends of the interaction region. This is reflected in the factor $\exp[-2z^2\theta_x^2/\sigma_x^2]$, and similarly for y, which depends on the crossing angle and the transverse emittances. (There are other factors which can affect the length of the luminous region such as cogging offset, orbit separations in the interaction region, and transverse coupling, but these are not considered in this memo.)

The longitudinal rms length of the luminous region is then given by the expression

$$\sigma_{\mathcal{L}z}^2 = \int \frac{z^2}{(1+z^2/\beta^{*2})} \exp[-\frac{z^2}{2\sigma_z^2} - \frac{2z^2}{\sigma_x^2}\theta_x^2 - \frac{2z^2}{\sigma_y^2}\theta_y^2]\, dz \Big/ \int \frac{1}{(1+z^2/\beta^{*2})} \exp[-\frac{z^2}{2\sigma_z^2} - \frac{2z^2}{\sigma_x^2}\theta_x^2 - \frac{2z^2}{\sigma_y^2}\theta_y^2]\, dz. \tag{4}$$

Equation 4 shows that length of the luminous region is about equal to σ_z whenever $\sigma_z << \beta^*$. When σ_z is larger than β^* however, the hourglass effect becomes important and the length of the luminous region is less than σ_z by a significant fraction.

This is shown in Figure 4 which plots the rms length of the luminous region, $\sigma_{\mathcal{L}z}$, as a function of σ_z for a β^* of 35 cm and a β^* of 100 cm. Note that the rms length of the luminous region begins to level off as the bunch length parameter σ_z becomes larger. To give perspective, we note that the length of a 53 MHz bucket is 564 cm, implying that a bunch length parameter of $\sigma_z = 100$ cm is quite large. Also shown in Figure 4 is the effect of a crossing angle on the length of the luminous region. This is shown for the crossing angle expected in later Run II with 132 nsec bunch spacing and with 95% normalized emittances of 20π mm-mrad and 40π mm-mrad. As seen from this plot the crossing angle significantly reduces the length of the luminous region.

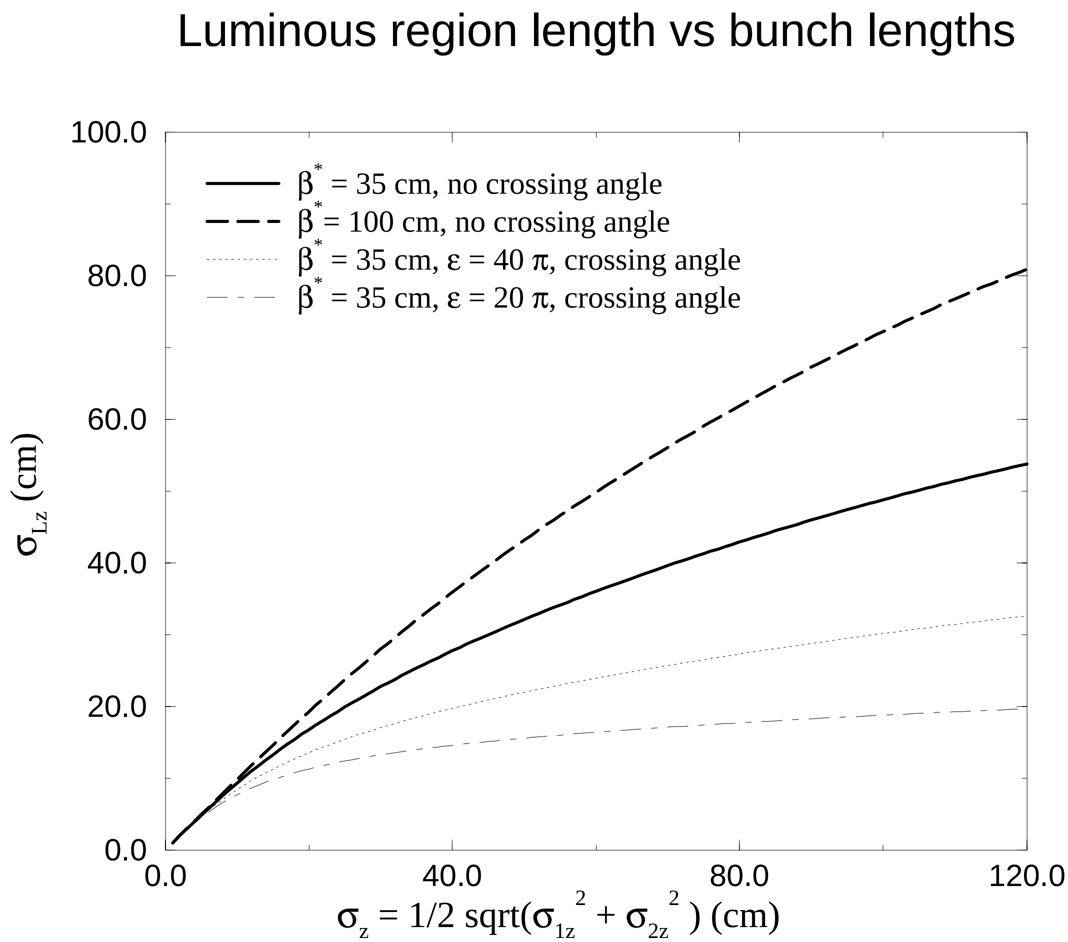

Figure 4: The rms length of the luminous region as a function of the rms bunch length parameter $\sigma_z = \sqrt{(\sigma_{1z}^2 + \sigma_{2z}^2)}/2$. The rms bunch lengths of the proton and antiproton bunches are σ_{1z} and σ_{2z}. The solid (dashed) line is for a β^* of 35 cm (100 cm) and no crossing angle. If there is no crossing angle then the transverse emittance does not affect the length of the luminous region. The dotted (dashed-dotted) line is for a $\beta^* = 35$ cm, a crossing half-angle of 136 μrad in each plane, and a 95% normalized transverse emittance of 40π mm-mrad (20π mm-mrad). In our definition the proton and antiproton closed orbits cross each other with an angle of 2×136 μrad in each plane. The total angle between the proton and antiproton closed orbits is 385 μrad.

Using Equation 4 and the calculated time evolution of beam parameters from Figures 1–3, we can calculate the rms length as a function of time in the store. This is done for antiproton intensities of 35, 70, and 100×10^9 antiprotons per bunch and is plotted in Figure 5. Because the bunch lengths begin to level off during a store, and because the length of the luminous region becomes less sensitive to bunch lengths as the store progresses, the length of the luminous region changes most rapidly in the beginning of the store.

Another, perhaps more useful, method to quantify the significance of the luminous region length is to calculate the "detector efficiency", ε_d. Since a collider detector has a limited range of acceptance in the longitudinal direction it is useful to know the fraction of events occurring within the acceptance. We use a very simple model of the acceptance and assume the detector can reconstruct 100% of the tracks with a vertex in the region $|z| < z_d$ and none of the tracks outside this region. With this simple model ε_d is equivalent to the fraction of luminosity occurring within the range $|z| < z_d$. In Figure 6 we plot the "detector efficiency" using the estimated Run II beam parameters and lifetimes plotted in Figures 1–3. ε_d is plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiprotons per bunch and for z_d values of 40, 50, and 60 cm. Deciding on the merit of increasing the detector acceptance is left as an exercise to the detector designers.

The typical length of a Run II store will be chosen to maximize the total integrated luminosity. There are many factors which will determine the optimum store length. These include shot setup time, antiproton recycling efficiency, stacking rate, maximum stack size, and Recycler performance. We do not explore the store length in this memo except to say that we expect the optimum store lengths to be 4 to 24 hours.

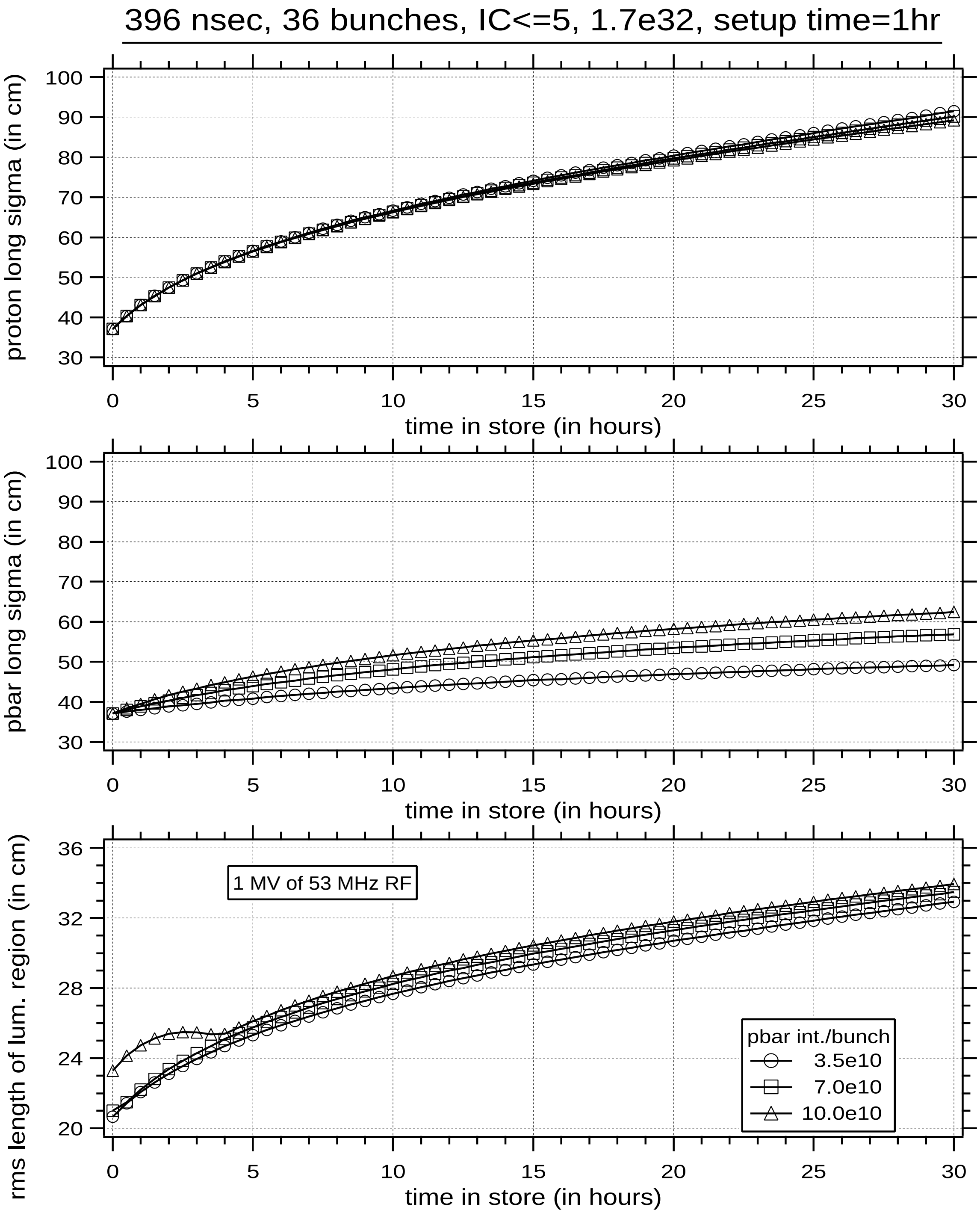

Figure 5: Estimated proton bunch length, antiproton bunch length, and rms length of the luminous region during a Run II store. These are plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiproton per bunch.

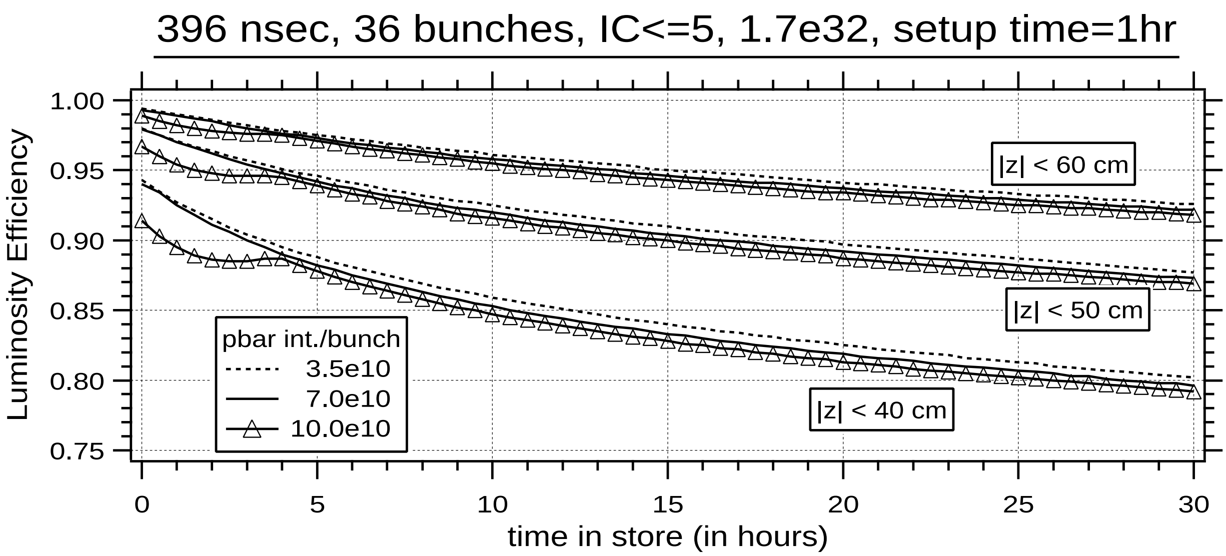

Figure 6: The “detector efficiency” during a Run II store. The “detector efficiency” is defined as the fraction of luminosity occurring inside a region $|z| < z_d$. This is plotted for initial antiproton bunch intensities of 35, 70 and 100×10^9 antiprotons per bunch and for z_d values of 40, 50, and 60 cm.

5 Transverse Luminosity Distribution

The width of the luminous region varies along the length of the interaction region due to the focusing of the beams. Therefore we present a formula for the rms width of the luminous region as a function of the longitudinal coordinate z. To simplify the expression we assume zero dispersion in the interaction region and $\alpha_x^* = \alpha_y^* = 0$. With these assumptions the horizontal rms width of the proton beam, for example, is given by

$$\sigma_{1x}(z) = \sqrt{\epsilon_{1x}\beta_x^*}\sqrt{1 + z^2/\beta_x^{*2}} \tag{5}$$

where ϵ_{1x} is the horizontal un-normalized emittance according to Equation 2.

The transverse luminosity distribution, derived in Appendix A, is the product of the proton and antiproton transverse distributions. With the usual assumption of Gaussian transverse distributions for the protons and antiprotons, the transverse distribution of the luminous region is also Gaussian. This results in a luminous region with a rms width, $\sigma_{\mathcal{L}x}$, which varies with the longitudinal position as

$$\frac{1}{\sigma_{\mathcal{L}x}^2} = \frac{1}{\sigma_{1x}^2} + \frac{1}{\sigma_{2x}^2}. \tag{6}$$

Combining this with the expression for the proton and antiproton beam widths in Equation 5, we get the simple expression

$$\sigma_{\mathcal{L}x} = \sqrt{\beta_x^* \frac{\epsilon_{1x}\epsilon_{2x}}{\epsilon_{1x} + \epsilon_{2x}}}\sqrt{1 + z^2/\beta_x^{*2}} \tag{7}$$

for the rms width of the luminous region. For example, if the 95% normalized horizontal emittances of the proton and antiproton beams are 20π mm-mrad, then $\epsilon_{1x} = \epsilon_{2x} = 3.13 \times 10^{-9}$ meters at an energy of 1 TeV. For a β^* of 35 cm this gives an rms width of

$$\sigma_{\mathcal{L}x}\ (\mu\text{meters}) = 23.4\sqrt{1 + z^2/\beta_x^{*2}}. \tag{8}$$

We note that the width of the luminous region is not changed by the introduction of a crossing angle or a separation of the proton and antiproton closed orbit. This is because luminosity width, which is the product of the proton and antiproton Gaussian profiles, does not change if the proton and antiproton closed orbits are separated. (It is possible for the center of transverse luminosity to shift if the proton and antiproton emittances are different. This effect is included in Equation 23 and the detailed derivations of Appendix A.)

A The Luminosity Integral

This appendix derives a formula relating beam parameters to the luminosity distribution in the interaction region. We begin with the general formula for luminosity and relate this to beam emittances by parameterizing the beam density distributions as Gaussians. By making several reasonable approximations a simplified version of the formula for calculating luminosity is derived and presented at the end of the appendix.

In this derivation we do not consider the possibility of transverse coupling between the horizontal and vertical planes. This assumption is not necessarily true as past Tevatron collider running has demonstrated. Formulas accounting for the effects of transverse coupling do exist [6] but we do not add this complication to our description of the luminosity since the design Tevatron lattice is not coupled.

The general formula for the luminosity of a single proton and antiproton bunch colliding in a circular accelerator is given by the overlap integral [3, 4]

$$\mathcal{L} = 2f_{\mathrm{rev}} \iiiint \rho_1(x, y, z, ct)\rho_2(x, y, z, ct)\, dx\, dy\, dz\, d(ct) \tag{9}$$

where f_{rev} is the revolution frequency of the circulating bunches, c is the speed of light, and $\rho(x, y, z, ct)$ is the particle density of the proton (subscript 1) or antiproton (subscript 2) bunch as a function of time. The z coordinate is the longitudinal coordinate (along the beam axis) with $z = 0$ nominally at the center of the detector and x and y are the transverse coordinates. The integrals are done for a single bunch crossing and over the entire interaction region.

Equation 9 is applicable to any particle distribution but is simplified and easily related to beam parameters if we assume Gaussian distributions for the particle densities. In the past a Gaussian distribution has been a good approximation of the actual beam distributions during normal Tevatron collider operations. We will return to Equation 9 but first we write down the expression for the transverse width of the proton or antiproton bunches in the interaction region.

Since the interaction region is a drift space with no magnetic fields[1] the lattice function, $\beta_x(z)$, is a quadratic function with two parameters

$$\beta_x(z) = \beta_x^* - 2\alpha_x^* z + \frac{1 + \alpha_x^{*2}}{\beta_x^*} z^2 \quad \text{or} \quad \beta_x(z) = \beta_{x,\mathrm{min}} + \frac{(z - z_{x,\mathrm{min}})^2}{\beta_{x,\mathrm{min}}} \tag{10}$$

where β_x^* and α_x^* are the values of the β and α functions[2] at $z = 0$. (The second form of the equation emphasizes that the minimum β value is not at $z = 0$ if $\alpha_x^* \neq 0$. In the design lattice of the Tevatron α^* is zero and the minimum β value is at the center of the detector, $z = 0$.)

If we assume a Gaussian distribution for the transverse phase space then the transverse beam profile will also be a Gaussian [7]. Temporarily ignoring the momentum spread and dispersion, the rms width of the bunch, σ_x, is related to the bunch emittance, ϵ_x, and the beta function, β_x, via

$$\sigma_x^2(z) = \epsilon_x \beta_x(z). \tag{11}$$

Here the emittance, ϵ, is related to the 95% normalized emittance usually quoted at Fermilab, $\epsilon_{N,95\%}$, via

$$\epsilon = \frac{\epsilon_{N,95\%}}{6\pi(\beta\gamma)_{\mathrm{rel}}} \tag{12}$$

[1] The magnetic field from the experimental solenoid has a negligible effect on the lattice functions.

[2] $\alpha(z) = -\frac{1}{2}(\frac{d\beta}{dz})$

where $(\beta\gamma)_{\rm rel}$ is the relativistic $\beta\gamma$. As an example consider the case of 20π mm-mrad 95% normalized emittance in the Tevatron at an energy of 1000 Gev. Then

$$\epsilon = \frac{\epsilon_{N,95\%}}{6\pi(\beta\gamma)_{\rm rel}} = \frac{20\pi \times 10^{-6}\text{mrad}}{6\pi(1000/0.938)} = 3.127 \times 10^{-9}\text{meters} \tag{13}$$

and the horizontal beam width at the interaction point where β_x^* is 35 cm is $\sigma_{1x} = \sqrt{\epsilon_{1x}\beta_x^*} = 33.1\mu$m.

The momentum spread of the particles in a bunch and the lattice dispersion function can also contribute to the transverse width of the beam. Compared to a reference momentum, p, a particle with a momentum offset $\frac{\Delta p}{p}$ will have a transverse closed orbit which is shifted by $\Delta x_{\rm c.o.}$. This closed orbit offset is related to the dispersion function, $\eta_x(z)$, via

$$\Delta x_{\rm c.o.} = \frac{\Delta p}{p}\eta_x(z). \tag{14}$$

Since the interaction region is a drift space with no magnetic fields, the dispersion is a linear function in z

$$\eta_x(z) = \eta_x^* + \eta_x'^* z \tag{15}$$

where η_x^* and $\eta_x'^*$ are the values of the dispersion and dispersion slope at $z = 0$.

The total transverse beam width is determined by combining the contributions of the beam emittance, ϵ_1, and the rms momentum spread, σ_{1p} [7]. If p is the average momentum of the particle beam then the transverse width is given by

$$\sigma_{1x}^2(z) = \epsilon_{1x}\beta_x(z) + \left(\frac{\sigma_{1p}}{p}\eta_x(z)\right)^2 \tag{16}$$

In collider Run I the dispersion had less than a 1% effect on the luminosity and during Run II the dispersion and its slope are expected to be zero in the interaction region. We only include the effect of dispersion in this appendix for completeness.

For the longitudinal distribution of the beam we assume a Gaussian distribution in the longitudinal phase space and a Gaussian profile with an rms width of σ_{1z}. The longitudinal bunch profile moves along the z-axis at the speed of light and the shape remains constant. For completeness we give the relationship between the longitudinal emittance and the bunch length when the bunch length is short compared to the length of the RF bucket.[3] The 95% normalized longitudinal emittance, $S_{N,95\%}$, and bunch length are related via

$$\sigma_{1z}^2 = \frac{c^2\beta_{\rm rel}}{12\pi^2 f_{\rm rev}}\sqrt{\frac{2\pi|\eta|}{heVE}}S_{1N,95\%} \tag{17}$$

where V is the RF voltage, $|\eta|$ is the slip factor, E is the energy of the beam, and h is the harmonic number of the Tevatron. For an RF voltage of 1 MV in the Tevatron at 1000 GeV this gives

$$\sigma_{1z} \ \ (\text{cm}) = 25.5 \times \sqrt{S_{1N,95\%} \ \ (\text{eVsec})} \tag{18}$$

(To prevent confusion we note that this is the length of the proton or antiproton bunch and is different from the length of the luminous region.)

By now combining the Gaussian longitudinal and transverse distributions of the beam we get the following distribution for the proton bunch

$$\rho_1(x,y,z,t) = \frac{N_1}{\sqrt{2\pi}^3\sigma_{1x}\sigma_{1y}\sigma_{1z}}\exp[-\frac{(x-\Delta x/2)^2}{2\sigma_{1x}^2} - \frac{(y-\Delta y/2)^2}{2\sigma_{1y}^2} - \frac{(z-ct)^2}{2\sigma_{1z}^2}] \tag{19}$$

[3]The bucket length is 565 cm in the Tevatron.

and for the antiproton bunch

$$\rho_2(x,y,z,t) = \frac{N_2}{\sqrt{2\pi}^3\sigma_{2x}\sigma_{2y}\sigma_{2z}} \exp[-\frac{(x+\Delta x/2)^2}{2\sigma_{2x}^2} - \frac{(y+\Delta y/2)^2}{2\sigma_{2y}^2} - \frac{(z+ct-ct_0)^2}{2\sigma_{2z}^2}] \tag{20}$$

where N_1 (N_2) is the number of protons (antiprotons) in the bunch. Remember that σ_{1x}^2 and σ_{1y}^2 are functions of z given in Equation 16. We have also added additional terms to this distribution to properly account for the cogging offset, orbit offset, and crossing angle of the proton and antiproton bunches.

First we discuss the cogging offset. By our definition the center of the proton bunch arrives at $z = 0$ (the center of the interaction region and ideally the center of the detector as well) at time $t = 0$. The antiproton bunch arrives at $z = 0$ at time $t = t_0$ due to a cogging offset. In other words the centers of the two bunches do not cross the center of the interaction region at the same time if there is a cogging offset. This implies that the centers of the two bunches collide at location $z = ct_0/2$ and time $t = t_0/2$.

The Δx and Δy are the horizontal and vertical distances between the proton and antiproton closed orbits which can arise from the use of electrostatic separators in the Tevatron. For given beam densities the luminosity will be a maximum if the two bunches collide head on and without a crossing angle. In this case $\Delta x = \Delta y = 0$. If the proton and antiproton orbits are not coincident then the offsets are linear functions of the longitudinal coordinate z,

$$\Delta x(z) = \Delta x_0 + 2z\tan\theta_x \quad \text{OR} \quad \Delta x(z) = \Delta x_0 + z\Delta x'. \tag{21}$$

The $\Delta x(z)$ is the horizontal separation of the proton and antiproton closed orbits as a function of z and θ_x is referred to as the crossing angle. In our definition the proton and antiproton closed orbits cross each other with an angle of $2\theta_x$. (When referring to the crossing angle it should be explicitly stated whether this refers to the change in proton closed orbit, or the angle between the proton and antiproton closed orbit.)

By substituting the distributions 19 and 20 into the overlap integral we can reduce Equation 9 to a single integral over the variable z. As an intermediate step we integrate over time and rearrange the expression to arrive at

$$\mathcal{L} = \frac{\sqrt{2\pi}N_1N_2f_{\text{rev}}}{2(2\pi)^3\sigma_z} \int\!\!\int\!\!\int \frac{dx\,dy\,dz}{\sigma_{1x}\sigma_{1y}\sigma_{2x}\sigma_{2y}} \exp[-\frac{(x-x_0)^2}{2\sigma_{\mathcal{L}x}^2} - \frac{(y-y_0)^2}{2\sigma_{\mathcal{L}y}^2} - \frac{(z-z_0)^2}{2\sigma_z^2} - \frac{\Delta x^2}{2\sigma_x^2} - \frac{\Delta y^2}{2\sigma_y^2}] \tag{22}$$

where

$$\begin{aligned}
\Delta x(z) &= \Delta x_0 + 2z\tan\theta_x \\
\sigma_z^2 &= (\sigma_{1z}^2 + \sigma_{2z}^2)/4 \\
\sigma_x^2(z) &= \sigma_{1x}^2(z) + \sigma_{2x}^2(z) \\
\frac{1}{\sigma_{\mathcal{L}x}^2} &= \frac{1}{\sigma_{1x}^2} + \frac{1}{\sigma_{2x}^2} \\
x_0(z) &= \frac{\Delta x}{2}\left(\frac{\sigma_{2x}^2 - \sigma_{1x}^2}{\sigma_{1x}^2 + \sigma_{2x}^2}\right) \\
z_o &= ct_0.
\end{aligned}$$

From this equation we can see that the transverse luminosity distribution is the product of the two transverse Gaussians of the bunch distributions. Thus the rms transverse width of the luminous region is simply $\sigma_{\mathcal{L}x}$. Note that the transverse width of the luminous region is a function of z but is independent of the crossing angle, orbit offset, or bunch length.

At this point we comment on the distinction between the transverse width of the beam and the transverse width of the luminous region. The transverse distribution of the luminous region is a Gaussian with an rms width of $\sigma_{\mathcal{L}x}$. It is the product of the Gaussian distributions of the proton and antiproton bunch widths and the rms value is derived from the overlap integral and . Thus

$$\frac{1}{\sigma_{\mathcal{L}x}^2} = \frac{1}{\sigma_{1x}^2} + \frac{1}{\sigma_{2x}^2} \tag{23}$$

Continuing with the previous example if both the protons and antiprotons have 95% normalized transverse emittances of 20π mm-mrad then the transverse width of the protons and antiprotons at the interaction point is 33.1 μm but the transverse width of luminous region at the interaction point is only $\sigma_{\mathcal{L}x} = 23.4$ μm.

Next we can integrate Equation 22 over x and y and arrive at a formula for calculating luminosity

$$\mathcal{L} = \frac{N_1 N_2 f_{\mathrm{rev}}}{2\pi} \frac{1}{\sqrt{2\pi}\sigma_z} \int \frac{1}{\sigma_x \sigma_y} \exp[-\frac{(z-z_0)^2}{2\sigma_z^2} - \frac{\Delta x^2}{2\sigma_x^2} - \frac{\Delta y^2}{2\sigma_y^2})]\, dz. \tag{24}$$

Note that even with the assumption of Gaussian distributions 23 parameters are needed to calculate the luminosity. They are

- proton and antiproton intensities N_1, N_2
- horizontal and vertical proton and antiproton emittances $\epsilon_{1x}, \epsilon_{1y}, \epsilon_{2x}, \epsilon_{2y}$
- proton and antiproton bunch lengths σ_{1z}, σ_{2z}
- proton and antiproton momentum spreads σ_{1p}, σ_{2p}
- lattice functions $\beta_x^*, \alpha_x^*, \eta_x^*, \eta_x'^*, \beta_y^*, \alpha_y^*, \eta_y^*, \eta_y'^*$
- cogging offset z_0
- horizontal and vertical orbit offset and crossing angles $\Delta x_0, \theta_x$, Δy_0, θ_y

The complexity of the luminosity equation can be reduced by making some reasonable assumptions and approximations. First we assume that the z locations of β^*, the detector, and the cogging point (z_0) are all at $z = 0$ and that the vertical and horizontal β^* values are the same. The dispersion is approximated as zero along the entire IR which is consistent with the design value of zero. We allow there to be a crossing angle between the proton and antiproton closed orbits ($\theta_z \neq 0$ and $\theta_y \neq 0$) but assume the orbits are not separated at the interaction point ($\Delta x_0 = \Delta y_0 = 0$.) We also use the small angle approximation $\theta_x \ll 1$ and $\theta_y \ll 1$. Finally we assume that the vertical and horizontal emittances of each species of particles are the same ($\epsilon_{1x} = \epsilon_{1y} = \epsilon_1$). With these approximations, which eliminates 14 parameters, the expression for luminosity simplifies to

$$\mathcal{L} = \frac{f_{\mathrm{rev}} N_1 N_2}{2\pi\beta^*(\epsilon_1 + \epsilon_2)} \frac{1}{\sqrt{2\pi}\sigma_z} \int \frac{1}{(1 + z^2/\beta^{*2})} \exp[-\frac{z^2}{2\sigma_z^2} - \frac{2z^2}{\sigma_x^2}\theta_x^2 - \frac{2z^2}{\sigma_y^2}\theta_y^2]\, dz. \tag{25}$$

The simplified formula is determined by the 9 parameters, N_1, N_2, ϵ_1, ϵ_2, β^*, σ_{1z}, σ_{2z}, θ_x, and θ_y.

B Run I Beam Parameters

Figure 7 shows a series of histograms of beam parameters during Run I operations. The data represent all the collider running from March 8, 1995 through April 21, 1995 except for a few stores where the data set was unavailable or internally inconsistent. All the parameters were obtained from data which was periodically collected during the store during the interval from one to five hours after the beams achieved collisions at low beta. A more detailed history of Run I performance can be found in reference [5].

For our purposes we display the histograms to give the reader an idea of the variation of beam parameters and luminosity from store to store. (Since the data in the histograms were actually taken during the interval from one to five hours after the beam achieved collision, they are not the variations of the parameters at the start of the store. However the data plotted was conveniently available and demonstrates the range of beam parameters expected during the beginning hours of a store.) The largest variations are the antiproton intensity and the luminosity. The antiproton intensity varies by a factor of 3 and is the major contribution to the factor of 5 variation in the luminosity. The antiproton intensity variation is largely caused by Tevatron and Accumulator failures which resulted in a large variation of the number of antiprotons available in the Accumulator at the start of a shot setup.

Also of interest is the growth rate of the emittances and bunch lengths during a store and the beam intensity lifetime. Figures 8–11 show the time evolution of the luminosity, beam intensity, longitudinal emittance, and transverse emittance during a typical store in Collider Run I. The points in these plots are the measured data and the solid lines are calculations based on concepts such as particle loss from collisions, intrabeam scattering, and residual gas scattering [1]. As seen in the plots the beam evolution models have only modest success but we still use them to make projections for Run II. It should be noted that Run II will differ significantly from Run I due to the increased number of parasitic crossings. We have only limited experience with this type of operation and the effects that long range beam-beam interactions have on the beam lifetimes is not well understood.

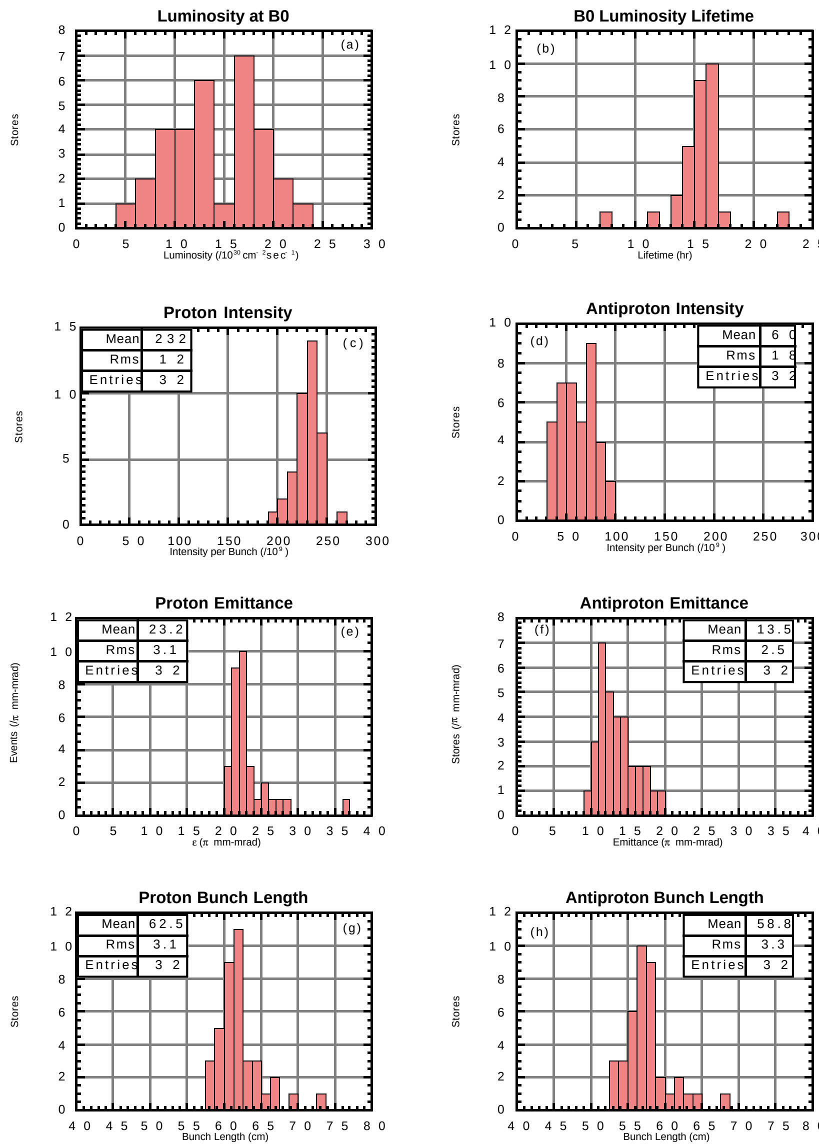

Figure 7: Summary of luminosity and beam parameters during routine Run I operations.

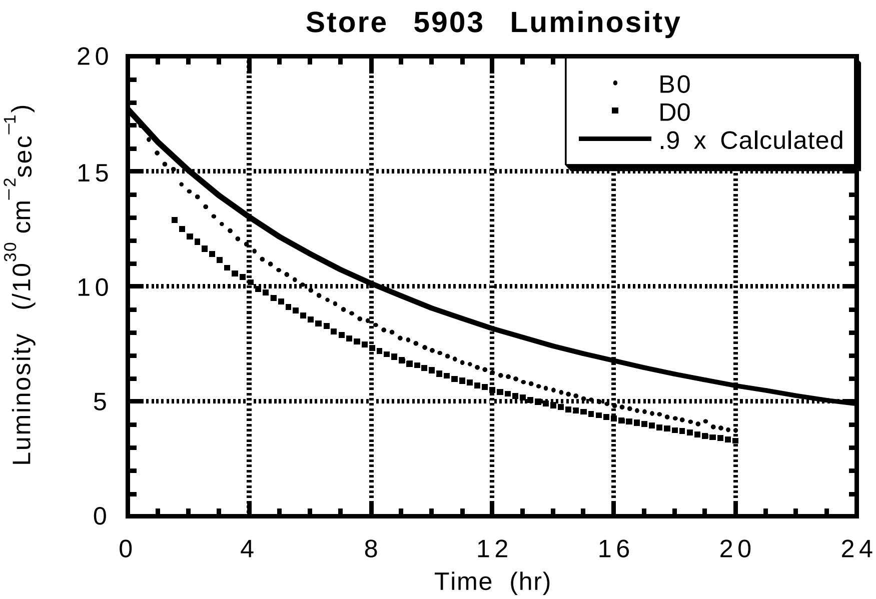

Figure 8: Luminosity during the evolution of a Run I store. The dotted lines are the measured luminosities at B0 and D0. The solid line is the luminosity calculated by the Beams Division based on measured beam parameters. The calculated value is multiplied by an ad-hoc factor of 0.9 to make it agree with the B0 measurement at the start of the store.

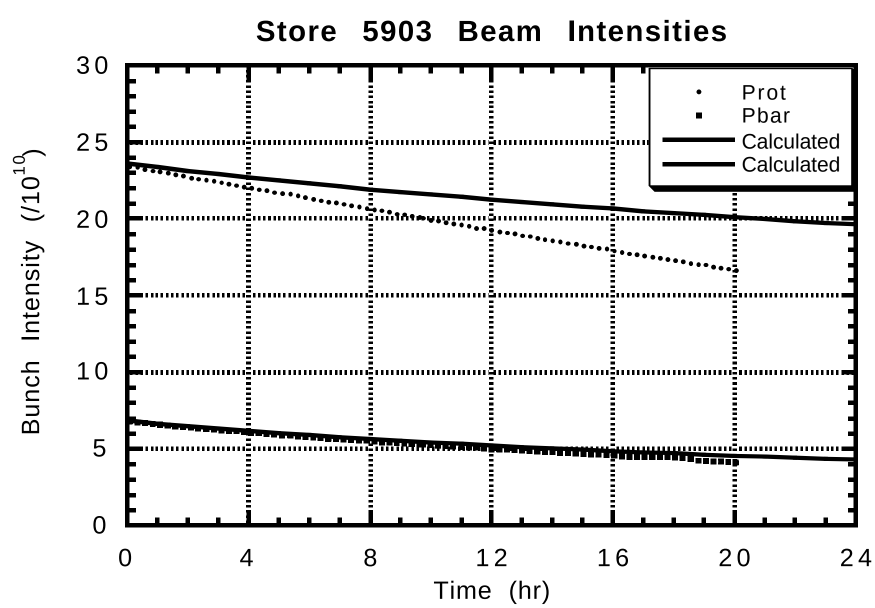

Figure 9: Intensity of the proton and antiproton bunches during the evolution of a Run I store. The dotted lines are the measured intensities and the solid lines are calculated values based on lifetime models.

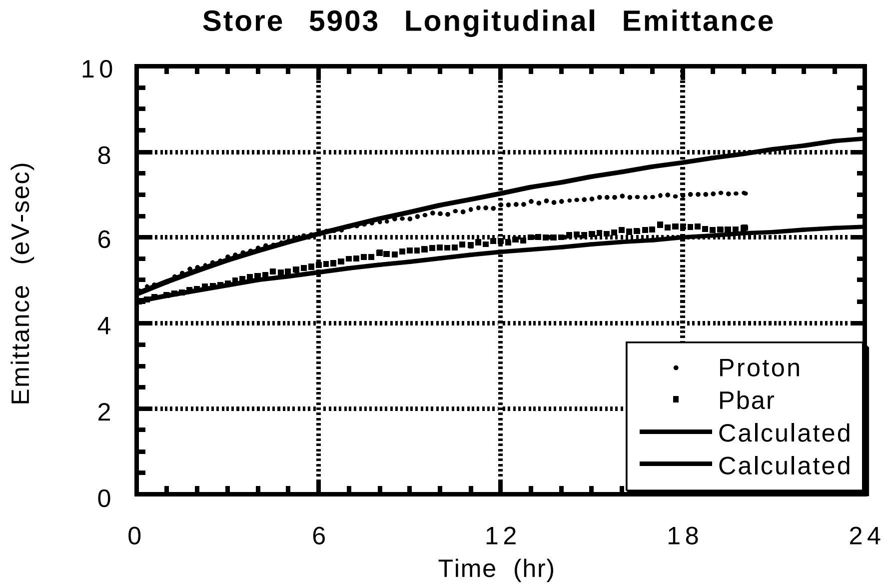

Figure 10: Longitudinal 95% normalized emittance of the proton and antiproton bunches during the evolution of a Run I store. The dotted lines are the measured longitudinal emittances and the solid lines are calculated values based on lifetime models.

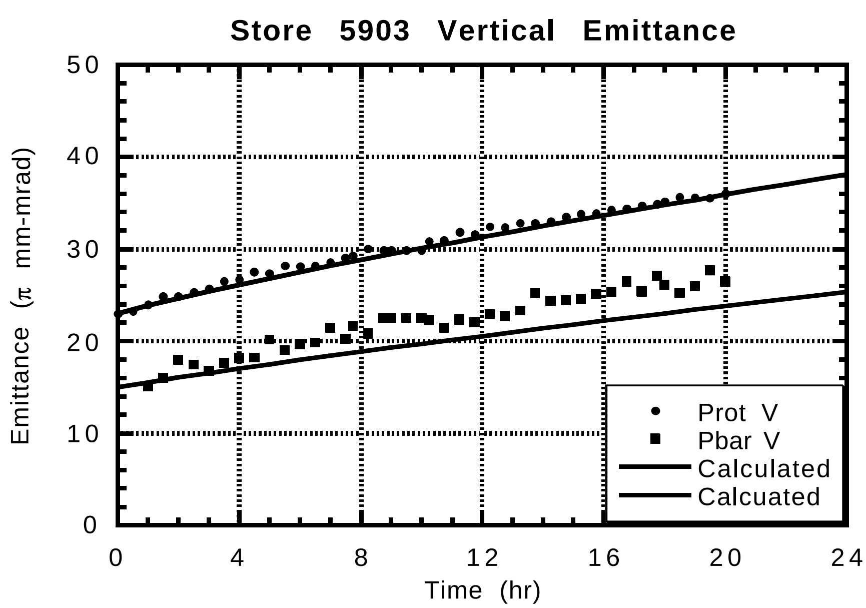

Figure 11: Vertical 95% normalized emittance of the proton and antiproton bunches during the evolution of a Run I store. The dotted lines are the measured vertical emittance and the solid lines are calculated values based on lifetime models.

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